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Refined H^∞-Optimal Approach to Rotorcraft Flight Control

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This paper introduces an approach to refine the H^∞-optimal controller for the four-block H^∞ control problem. The second singular value of the compensated system can be analyzed and synthesized with a free parameter by the proposed approach. This approach is implementable in computation with an appropriate selection of a diagonalizing matrix pair. The H^∞ norm of the sublayers can, therefore, be improved. The characterization of the sublayers for the four-block problem is also completed. The problems that required higher robustness are suggested by this proposed approach. Furthermore, an engineering application concerning the rotorcraft flight control is provided. The simulation results bestow a promising progress both in the frequency domain and in the time domain. The gain margin, phase margin, settling time, and damping effect are improved in this example.

Introduction

THE helicopter is a kind of aircraft known as a rotorcraft since the necessary lift to sustain flight is produced by means of the rotating wings, the rotors. Recently, a number of papers, which discussed the application of various modern feedback control design techniques to helicopter flight control synthesis, have appeared. Helicopter dynamics are characterized by multi-input/multi-output (MIMO) mathematical models since helicopter responses to control inputs are highly coupled. Modern MIMO techniques are well suited to the design of control laws for helicopters, and numerous papers have described such applications. These include linear quadratic regulator theory,[1,2] model following,[3] optimal output feedback design,[4] H^∞ techniques,[5] and eigenstructure assignment.[6]

In this decade, the H^∞ control problem has become one of the dominant streams in the realm of control systems since the pioneering work of Zames.[7] The original solution of the H^∞-norm minimization problem obtained by solving an equivalent model matching problem is summarized by Francis[8] along with the relevant factorization theory for an excellent introduction to this subject. A number of different approaches have been investigated. Recently, significant progress has been made by taking a state-space approach. The H^∞-optimal controller synthesis problem has been shown to lead to the solution of two algebraic Riccati equations. Complete state-space solutions for all stabilizing controllers that simultaneously minimize the H^2 norm with constraints in the H^∞ norm of the appropriate closed-loop transfer function are given in Ref. 9 and the references therein for this state-space approach.

The standard problem in controller design can be formulated by linear fractional transformation as

$$T_{wv} = LFT(P, -K) \equiv P_{11} - P_{12}K(I + P_{22}K)^{-1}P_{21} \tag{1}$$

mathematically[8] (shown in Fig. 1) where T_{wv} is the transfer function from v to w, v is the exogenous input, u is the control signal, w is the controlled output, and y is the measurement output, whereas

$$P \equiv \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

with $P_{11} \in RL^\infty_{(p_1+p_2)\times(q_1+q_2)}$, $P_{12} \in RL^\infty_{(p_1+p_2)\times p_1}$, $P_{21} \in RL^\infty_{q_1\times(q_1+q_2)}$, and $P_{22} \in RL^\infty_{q_1\times p_1}$. Conventionally, the parameterization is employed involving coprime factorization. The standard problem becomes the equivalent model matching problem as follows:

$$T_{wv} = T_1 - T_2QT_3 \tag{2}$$

where T_1, T_2, and T_3 are with compatible dimensions, respectively. See Fig. 2. From this transformation with unitary invariance,[10] the H^∞-optimal control problem can then yield to an equivalent general distance problem (GDP) in H^∞,[8,11,12] i.e.,

$$\inf_{Q \in RH^\infty_+} \left\| \begin{bmatrix} R_{11} - Q & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \right\|_\infty = \gamma_{\text{opt}} \tag{3}$$

where R_{11}, R_{12}, R_{21}, and R_{22} are all in RL^∞ with the dimensions $p_1 \times q_1$, $p_1 \times q_2$, $p_2 \times q_1$, and $p_2 \times q_2$, respectively. For special cases when one of $(R_{21}\ \ R_{22})$ or

$$\begin{bmatrix} R_{12} \\ R_{22} \end{bmatrix}$$

is identically zero, the GDP becomes the so-called two-block problem (Hankel$\oplus$Toeplitz operator); although both of them are zero, the GDP degenerates to the one-block problem (Hankel operator). The optimal solution of Eq. (3) is not unique for a MIMO system.[13-15] There were no discussions about the sublayers in the H^∞-optimal control problem until the elegant concept of the superoptimization was proposed by Young.[16] The H^∞ norm of the sublayers may sometimes be as large as the optimal H^∞ norm of the major layer because of a poor choice of the optimal solutions, the nonlinear behavior, and the parameter variations of the plant if the plant is not good enough, inherently. The sublayer singular values reveal the gain in energy for all of the input-output patterns except the first one. The synthesis of the sublayers thereupon becomes interesting in the H^∞-optimal control problem. In this

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paper, the synthesis of the sublayers that refines the optimal solution of the major layer is considered. The maximal Schmidt pair of the four-block problem, which was introduced and constructed by Young and Lin,[17] inspires the authors to complete the crucial idea in the four-block problem, while the two-block problem was studied in Ref. 18. Here the diagonalizing matrix pair of the four-block problem is constructed. An implementable approach to synthesize the sublayers is also proposed with proper selection of the diagonalizing matrix pair. The McMillan degree of the refined H^∞-optimal controller need not be inflated. Also, the synthesis of the refined H^∞-optimal control for the flight control of a helicopter is discussed. The simulation results show that the performance is improved both in the frequency domain and in the time domain. In this paper, $RL^\infty(RL^\infty_{p\times q})$ denotes the space of the proper real rational matrix (of dimension $p \times q$) with no poles on the $j\omega$ axis including the point at ∞; RH^∞_+ $(RH^\infty_{+,p\times q})$ and RH^∞_- $(RH^\infty_{-,p\times q})$ are the subspaces of RL^∞ $(RL^\infty_{p\times q})$ without any poles in the closed right-half plane (RHP) and left-half plane (LHP), respectively; Ξ denotes the optimal solution set, or equivalently

$$\Xi \equiv \left\{ Q \in RH^\infty_+ \;\middle|\; \left\| \begin{bmatrix} R_{11}-Q & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \right\|_\infty = \gamma_{\text{opt}} \right\}$$

$G^*(s) = G^T(-s)$ is the para-Hermitian conjugate of $G(s)$, and

$$G(s) \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

means $G(s) = C(sI - A)^{-1}B + D$, where

$$s_j^\infty[G(s)] \equiv \sup_{\omega\in R} s_j[G(j\omega)]$$

and $s_j(\cdot)$ denotes the jth singular value of a matrix, especially $s_1^\infty(G) = \|G(s)\|_\infty$. From now on we may write G instead of $G(s)$, thereby taking the s dependence of transfer function matrix as implied.

Problem Formulation and Preliminary

The problem can be formulated as follows: Find a controller K such that

P1 T_{wv} is internally stable

P2 $\|T_{wv}\|_\infty$ is optimal

P3 $s^2_\infty(T_{wv}) \le \gamma' \le \gamma_{\text{opt}}$

where γ' is the performance bound of the sublayers.

The problem with P1 and P2 is the standard H^∞-optimal control problem. It is not easy to take P3 under consideration with P1 and P2 simultaneously, especially for the four-block problem. Conventionally, the following two assumptions are made for the four-block H^∞-optimal control problem:

A1 $\gamma_{\text{opt}} > \max\{\|(R_{21}\;\; R_{22})\|_\infty, \|(R_{12}^*\;\; R_{22}^*)\|_\infty\}$

A2 γ_{opt} is attainable within allowable accuracy tolerance after some iteration schemes.

In A1, the GDP in Eq. (3) that is strictly monotonically decreasing and convex in γ_{opt} is considered here. The optimal norm γ_{opt} can be calculated by the so-called γ iteration. A detailed treatment of the γ iteration is contained in Ref. 11. The numerical scheme is not discussed here. Accordingly, A2 is made. For the convenience of derivation, without loss of generality, and because of these assumptions, the normalized four-block problem,

$$\inf_{Q\in RH^\infty_+} \left\| \begin{bmatrix} R_{11}-Q & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \right\|_\infty = 1 \tag{4}$$

($\gamma_{\text{opt}} = 1$) is tackled in this paper except for the extraordinary description of it.

Lemma 1[17]: There exist two vectors v and w that are the maximal Schmidt pair of Eq. (4) such that the following equations

$$\begin{bmatrix} R_{11} - Q_{\text{opt}} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v = w(-s) \tag{5}$$

$$w^T \begin{bmatrix} R_{11} - Q_{\text{opt}} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = v^* \tag{6}$$

hold where $Q_{\text{opt}} \in \Xi$. □

These two vectors v and w are termed the maximal Schmidt pair, which reflects the input-output pattern in the situation of the highest gain in energy. For the robust problem in H^∞, this pair describes a destabilizing uncertainty. The details of the definition and construction of this pair are found in Ref. 17.

Lemma 2[19]: A matrix $x(s)$ in $RL^\infty_{p\times q}(s)$, where $p \ge q$. Then $x(s)$ can be factorized as

$$x(s) = x_i(s)x_o(s)x_c(s)$$

in which $x_i(s)$ is an inner function matrix in $RH^\infty_{+,p\times q}$, $x_o(s)$ is an outer function matrix in $RH^\infty_{+,q\times q}$, and $x_c(s)$ is in $RH^\infty_{-,q\times q}$, which is the so-called inverse lossless conjugator of $x(s)$. □

In general, both the vectors v and w in Lemma 1 may be in RL^∞. From Lemma 2, these two column vectors can, therefore, be factorized as follows:

$$v = v_i v_o v_c \qquad \text{and} \qquad w = w_i w_o w_c \tag{7}$$

□

Design Procedures

Let $v_\perp$ be the inner extension of v_i and $w_\perp$ be the inner extension of w_i. Construct and divide the matrices V and W as follows:

$$V_i \equiv (v_i\;\; v_\perp) \equiv \begin{bmatrix} v_{i,1} & v_{\perp,1} \\ v_{i,2} & v_{\perp,2} \end{bmatrix}$$

and

$$W_i \equiv (w_i\;\; w_\perp) \equiv \begin{bmatrix} w_{i,1} & w_{\perp,1} \\ w_{i,2} & w_{\perp,2} \end{bmatrix}$$

where $v_{i,1}$ is in $RH^\infty_{+,q_1\times 1}$, $v_{i,2}$ is in $RH^\infty_{+,q_2\times 1}$, $v_{\perp,1}$ is in $RH^\infty_{+,q_1\times(q_1+q_2-1)}$, and $v_{\perp,2}$ is in $RH^\infty_{+,q_2\times(q_1+q_2-1)}$, whereas $w_{i,1}$ is in $RH^\infty_{+,p_1\times 1}$, $w_{i,2}$ is in $RH^\infty_{+,p_2\times 1}$, $w_{\perp,1}$ is in $RH^\infty_{+,p_1\times(p_1+p_2-1)}$, and $w_{\perp,2}$ is in $RH^\infty_{+,p_2\times(p_1+p_2-1)}$. These two vectors V_i and W_i are one of the diagonalizing matrix pair.[19] Furthermore, construct ξ_i that is inner in $RH^\infty_{-,(p_1+p_2-1)\times(p_1-1)}$ with

$$w_{\perp,2}(-s)\xi_i(s) = 0 \tag{8}$$

and construct $\xi_\perp$ that is the inner extension of ξ_i. Also, construct η_i that is inner in $RH^\infty_{-,(q_1-1)\times(q_1+q_2-1)}$ with

$$\eta_i v^*_{\perp,2} = 0 \tag{9}$$

and construct $\eta_\perp$ that is the co-inner extension of η_i. Define the matrices as follows:

$$V(s) \equiv V_i(s)\begin{bmatrix} 1 & 0 \\ 0 & \eta^*(s) \end{bmatrix} \equiv V_i(s)\begin{bmatrix} 1 & 0 & 0 \\ 0 & \eta_i^*(s) & \eta_\perp^*(s) \end{bmatrix} \tag{10}$$

$$W(s) \equiv W_i(s)\begin{bmatrix}1 & 0\\ 0 & \xi(-s)\end{bmatrix} \equiv W_i(s)\begin{bmatrix}1 & 0 & 0\\ 0 & \xi_i(-s) & \xi_\perp(-s)\end{bmatrix} \tag{11}$$

Proposition 3: The matrices V and W defined in Eqs. (10) and (11) are one of the diagonalizing matrix pair of Eq. (4).

Proof: See Appendix A.

Theorem 4: $\hat{Q}_{\rm opt} \in \Xi$ satisfies

$$s_2^\infty\begin{bmatrix}R_{11}-\hat{Q}_{\rm opt} & R_{12}\\ R_{21} & R_{22}\end{bmatrix} \le \gamma'$$

if and only if

$$\hat{Q}_{\rm opt} = Q_{\rm opt} + w_{\perp,1}(-s)\xi_i Q'\eta_i v^*_{\perp,1} \tag{12}$$

with $w_{\perp,1}(-s)\xi_i Q'\eta_i v^*_{\perp,1} \in RH^\infty_+$, where $Q_{\rm opt}$ is any solution in Ξ and

$$s_1^\infty\begin{bmatrix}\bar{R}'_{11}-Q' & \bar{R}'_{12}\\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix} \le \gamma' \le 1 \tag{13}$$

where

$$\begin{bmatrix}\bar{R}'_{11} & \bar{R}'_{12}\\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix} \equiv \begin{bmatrix}\xi_i^* w^T_{\perp,1} & 0\\ \xi^*_\perp w^T_{\perp,1} & \xi^*_\perp w^T_{\perp,2}\end{bmatrix}\begin{bmatrix}R_{11}-Q_{\rm opt} & R_{12}\\ R_{21} & R_{22}\end{bmatrix} \times \begin{bmatrix}v_{\perp,1}\eta_i^* & v_{\perp,1}\eta^*_\perp\\ 0 & v_{\perp,2}\eta^*_\perp\end{bmatrix}$$

Proof: See Appendix B.

The optimal solution $Q_{\rm opt} \in \Xi$ can be calculated in each layer, and $g(s)$ is independent of $Q_{\rm opt}$, whereas

$$\begin{bmatrix}\bar{R}'_{11} & \bar{R}'_{12}\\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix}$$

is in RL^∞ varying with different $Q_{\rm opt}$. The sublayers of the four-block problem have been characterized by another four-block problem in this theorem. It is evident that both the analysis and synthesis of these sublayers are, of course, a four-block problem with the matrix dimensions reduced by 1, both the column and the row, compare Eq. (13), i.e., $\bar{R}'_{11}$ is in $RL^\infty_{(p_1-1)\times(q_1-1)}$, $\bar{R}'_{12}$ is in $RL^\infty_{(p_1-1)\times q_2}$, $\bar{R}'_{21}$ is in $RL^\infty_{p_2\times(q_1-1)}$, and $\bar{R}'_{22}$ is in $RL^\infty_{p_2\times q_2}$. From Eq. (12), the optimal solution $\hat{Q}_{\rm opt}$, which may not only preserve the optimal H^∞ norm of the major layer but also may satisfy some specifications in the sublayers, can be obtained from any optimal solution $Q_{\rm opt}$ solved in Eq. (4) by adding $w_{\perp,1}(-s)\xi_i Q'\eta_i v^*_{\perp,1}$. The problem for the sublayers becomes [with $w_{\perp,1}(-s)\xi_i Q'\eta_i v^*_{\perp,1} \in RH^\infty_+$]

$$\left\|\begin{bmatrix}\bar{R}'_{11}-Q' & \bar{R}'_{12}\\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix}\right\|_\infty \le \gamma' \tag{14}$$

where γ' is the requirement for the second singular value of Eq. (4).

The problem in Eq. (14) is not a standard GDP in H^∞. It cannot be implementable in computation to solve Q'. The problem in Eq. (14) should, therefore, be modified. Since the diagonalizing matrix pair is not unique, let the diagonalizing matrix pair be

$$\hat{V}(s) \equiv V_i(s)\begin{bmatrix}1 & 0 & 0\\ 0 & \eta_i^*(s)\hat{v}_c(-s) & \eta^*_\perp(s)\end{bmatrix} \tag{15}$$

$$\hat{W}(s) \equiv W_i(s)\begin{bmatrix}1 & 0 & 0\\ 0 & \xi_i(-s)\hat{w}_c(-s) & \xi_\perp(-s)\end{bmatrix} \tag{16}$$

where $\hat{v}_c$ and $\hat{w}_c$ are lossless conjugators of $v_{\perp,1}(-s)\eta_i^T$ and $w_{\perp,1}(-s)\xi_i$, respectively, i.e.,

$$v_{\perp,1}(-s)\eta_i^T = \hat{v}\hat{v}_c^{-1}$$

$$w_{\perp,1}(-s)\xi_i = \hat{w}\hat{w}_c^{-1}$$

It is not difficult to show that both $\hat{v}$ and $\hat{w}$ are the so-called minimal-phase inner functions without any transmission zeros in the left-half plane[20] and with compatible dimensions, respectively. It is easy to yield that

$$\hat{Q}_{\rm opt} = Q_{\rm opt} + \hat{w}Q'\hat{v}^T \tag{17}$$

whereas (with $Q' \in RH^\infty_+$)

$$\left\|\begin{bmatrix}R'_{11}-Q' & R'_{12}\\ R'_{21} & R'_{22}\end{bmatrix}\right\|_\infty \le \gamma' \tag{18}$$

where

$$\begin{bmatrix}R'_{11} & R'_{12}\\ R'_{21} & R'_{22}\end{bmatrix} \equiv \begin{bmatrix}\hat{w}^* & 0\\ \xi^*_\perp w^T_{\perp,1} & \xi^*_\perp w^T_{\perp,2}\end{bmatrix}\begin{bmatrix}R_{11}-Q_{\rm opt} & R_{12}\\ R_{21} & R_{22}\end{bmatrix} \times \begin{bmatrix}\hat{v}(-s) & v_{\perp,1}\eta^*_\perp\\ 0 & v_{\perp,2}\eta^*_\perp\end{bmatrix} \tag{19}$$

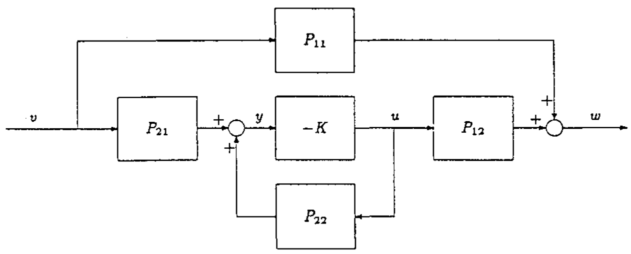

Fig. 1 Standard problem.

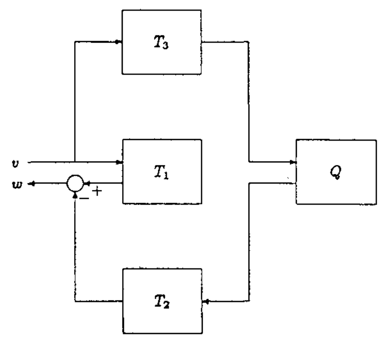

Fig. 2 Equivalent model matching problem.

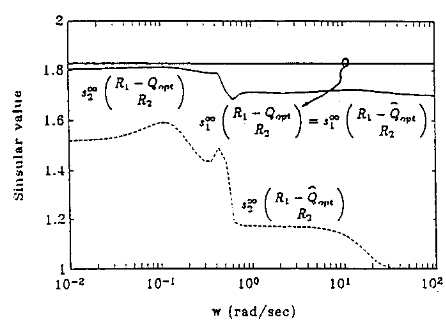

Fig. 3 First two singular value decomposition plots of these two compensated systems.

Table 1 State-space matrices of the Sikorsky S-61 helicopter

$A =$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
	0.0000	0.0000	−0.0420	0.3200	0.0030	0.0010
	0.0000	0.0000	−1.2300	−1.6000	0.0040	−0.0120
	−32.2000	0.0000	4.7000	−1.0000	−0.0200	−0.0050
	0.0000	32.2000	−1.0000	−4.7000	0.0050	−0.0200
$B =$	0.0000	0.0000	—	—	—	—
	0.0000	0.0000	—	—	—	—
	−0.3000	6.3000	—	—	—	—
	−23.0000	−1.1000	—	—	—	—
	1.0000	−32.2000	—	—	—	—
	−32.2000	1.0000	—	—	—	—
$C =$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$D =$	0.0000	0.0000	—	—	—	—
	0.0000	0.0000	—	—	—	—
	1.0000	0.0000	—	—	—	—
	0.0000	1.0000	—	—	—	—

Table 2 Poles of G, K_{opt}, and $\hat{K}_{opt}$

$G(s)$	K_{opt}	$\hat{K}_{opt}$
−1.4924	−33.0659	−41.6288
−0.6390	−0.0776	−4.0627
$0.0585 \pm 0.5049i$	−0.0582	−0.8648
$0.1662 \pm 0.3751i$	1.0930	$-0.0690 \pm 0.0041i$
—	6.7485	—

Table 3 State-space matrices of the controller K_{opt} with Q_{opt}

$A =$	−0.2272	0.0620	−0.6047	−0.1722	4.2053
	0.0875	−0.2224	−0.0229	−1.9174	18.0922
	0.3581	−0.0456	0.1330	−7.0076	16.6347
	0.0000	−0.0629	1.3550	8.8717	−36.7524
	0.0000	0.2847	0.2992	1.2251	−33.9152
$B =$	−1.0112	−0.2837	1.3722	−0.2484	—
	−9.4916	−8.4051	7.4462	−1.3481	—
	−14.6873	−16.8549	10.5480	−1.9097	—
	26.6818	27.2670	−19.6443	3.5565	—
	17.6230	9.2987	−11.9723	2.1675	—
$C =$	0.0000	0.0000	0.0000	−0.5642	2.1595
	0.0800	0.0000	0.0000	0.4266	2.8563
$D =$	−1.5138	−1.4997	1.1061	−0.2002	—
	−0.8490	0.3170	0.5859	−0.1061	—

Any of $Q_{opt} \in \Xi$ can generate $\hat{Q}_{opt}$ from Eq. (17) with $Q' \in RH^{\infty}_{+}$, which is the suboptimal solutions of the standard GDP in H^{∞} of the sublayers with the performance bound γ' [compare Eq. (18), of course $\gamma' \leq 1$], i.e., the H^{∞} norm of the sublayers can be improved by regulating $Q' \in RH^{\infty}_{+}$ in Eq. (18). In conclusion, the design procedure would be as follows.

Step 1: Parameterize the controller K by Q.

Step 2: Solve Q_{opt} from Eq. (4).

Step 3: Solve $\hat{Q}_{opt}$ from Eq. (17) with Eqs. (18) and (19).

Step 4: Recover the parameterization from the parameter $\hat{Q}_{opt}$ to the controller $\hat{K}_{opt}$.

The details of the parameterization are found in Refs. 8, 10, and 12. The $\hat{Q}_{opt}$ can be solved by Q_{opt} and Q' in Eq. (17). The state-space formula of K_{opt} can be obtained by Q_{opt}.[8,10-12,14] Furthermore, the state-space descriptions of $\hat{K}_{opt}$ can also be obtained by the same procedures as K_{opt}. All of the procedures need a great number of the state-space operations of pole-zero cancellations. □

Rotorcraft Flight Controller Synthesis

The helicopter model here is a Sikorsky S-61 helicopter. The control laws are designed for a sixth-order rigid-body model that is in hover flight condition with the rotor disk tilted instantaneously. The linearized rigid-body equations of motion are expressed in the standard state variable form as follows:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

where y is the measured output vector. Let

$$G(s) \stackrel{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

The appropriate values of the A, B, C, and D matrices are given in Table 1.[21] The state vector, input vector, and measured output vector are

$$x \equiv (\theta_F \;\; \phi_F \;\; q_F \;\; p_F \;\; u \;\; v)^T$$

$$u_c \equiv \begin{pmatrix} -\phi_R \\ \theta_R \end{pmatrix}$$

and

$$y \equiv (q_F \;\; p_F \;\; -\phi_R \;\; \theta_R)^T$$

where θ_F is the pitch attitude of the fuselage, ϕ_F the roll angle of the fuselage, q_F the pitch rate of the fuselage, p_F the roll rate of the fuselage, u the velocity along the x axis of the fuselage, and v the velocity along the y axis of the fuselage, whereas θ_R is the pitch (longitudinal) tilt angle of the rotor, and ϕ_R the roll (lateral) tilt angle of the rotor.

The eigenvalues of the given plant corresponding to the A matrix in Table 1 are shown in the first column of Table 2. There are four modes involving two stable subsidence (overdamped) modes and two unstable oscillation modes. The states of this helicopter will diverge because of a small disturbance even though all of the states are in equilibrium simultaneously. There should be a control strategy to deal with the given plant. The proposed approach in this paper is a candidate utilized to stabilize the unstable helicopter; also the results by the proposed method will be compared with those from the conventional approach for the H^{∞}-optimal controller.

The unweighted mixed output sensitivity H^{∞} control problem is considered since the complementary sensitivity function $T(s)$ reveals the robustness performance of the compensated system[22] whereas the sensitivity function $S(s)$ reflects the performance of the closed-loop system in frequency domain. By definition

$$T(s) \equiv -G(s)K(s)[I + G(s)K(s)]^{-1}$$

$$S(s) \equiv I - G(s)K(s)[I + G(s)K(s)]^{-1}$$

where $K(s)$ is the controller to be designed. The mixed sensitivity problem in H^∞-optimal control becomes

$$\inf_{K(s)} \|T_{wv}\|_\infty \equiv \inf_{K(s)} \left\| \begin{bmatrix} T(s) \\ S(s) \end{bmatrix} \right\|_\infty = \gamma_{\text{opt}} \tag{20}$$

or

$$\inf_K \| LFT(P, -K) \|_\infty = \gamma_{\text{opt}}$$

where

$$P(s) = \begin{bmatrix} \begin{pmatrix} O \\ I \end{pmatrix}_{8\times 4} & \begin{bmatrix} G(s) \\ G(s) \end{bmatrix}_{8\times 2} \\ (I)_{4\times 4} & [G(s)]_{4\times 2} \end{bmatrix}$$

whereas K stabilizes the plant G internally. From Eq. (20) and by coprime factorization, the equivalent distance problem in H^∞ becomes

$$\inf_{Q \in RH_+^\infty} \left\| \begin{pmatrix} R_{11} - Q \\ R_{21} \end{pmatrix} \right\|_\infty = \gamma_{\text{opt}} \tag{21}$$

This is a two-block problem with $R_{11} \in RL^\infty_{p_1 \times q}$, $R_{21} \in RL^\infty_{p_2 \times q}$, $R_{12} = 0$, and $R_{22} = 0$, where $p_1 = 2$, $p_2 = 6$, and $q = 4$. The McMillan degree of

$$\begin{pmatrix} R_{11} \\ R_{21} \end{pmatrix}$$

is 10, i.e., $n = 10$.

By γ-iteration scheme, the optimal H^∞ norm of Eq. (21) $\gamma_{\text{opt}} = 1.8288463407$ (the precision is 1.0×10^{-10}). The problem is continuous, strictly monotonically decreasing, and convex in γ_{opt} since $\gamma_{\text{opt}} > \|R_{21}\| = 1.0000$. One of the optimal solutions in Eq. (21) is Q_{opt} with McMillan degree equal to 9. The optimal controller corresponding to Q_{opt} is K_{opt} whose state-space description is listed in Table 3. The first two singular value plots of

$$\begin{pmatrix} R_{11} - Q_{\text{opt}} \\ R_{21} \end{pmatrix}$$

are shown in Fig. 3 where

$$s_2^\infty \begin{pmatrix} R_{11} - Q_{\text{opt}} \\ R_{21} \end{pmatrix} = 1.8111$$

Note that the second singular value is very close to the major singular value. It is supposed that the sublayers should be regulated.

Construct the diagonalizing matrix pair $\hat{V}(s)$ and $\hat{W}(s)$ as in Eqs. (15) and (16). Select the performance bound of the sublayers to be 1.6000, i.e.,

$$\left\| \begin{pmatrix} R_{11}' - Q' \\ R_{21}' \end{pmatrix} \right\|_\infty \leq 1.6$$

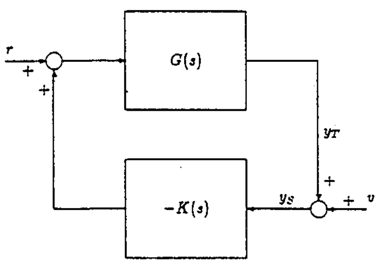

Fig. 4 Compensated system.

Table 4 State-space matrices of the controller $\hat{K}_{\text{opt}}$ with $\hat{Q}_{\text{opt}}$

$A =$	−0.1934	0.0424	0.0709	−0.6516	−8.0658
	0.0704	−0.0672	0.1591	−1.5296	9.8690
	0.3690	−0.0613	−1.1612	6.9159	−17.2903
	0.0000	0.2706	0.0459	−4.5238	2.5597
	0.0000	0.0127	−0.9779	6.6395	−40.7487
$B =$	−0.0091	−0.3472	−2.1226	−1.1244	—
	1.2995	0.3983	2.9906	1.5842	—
	−0.7307	8.3231	−9.5264	−5.0464	—
	0.4743	−7.0704	4.6479	2.4621	—
	0.6296	3.5778	−14.8998	−7.8928	—
$C =$	0.0000	0.0000	0.0000	−0.4153	1.6884
	0.0000	0.0000	0.0000	0.2606	2.6909
$D =$	0.0195	−0.4597	0.7809	0.4137	—
	−0.0367	0.8679	0.4137	0.2191	—

Table 5 Minimum singular value, gain margin, and phase margin of the compensated system

	Minimum singular value	Gain margin		Phase margin, deg
K_{opt}	0.6048	9.4597	18.5662	±35.2023
$\hat{K}_{\text{opt}}$	0.6155	9.5932	19.1173	±35.8487

where R_{11}' and R_{21}' are defined as in Eq. (19). From Eq. (17), the optimal solution $\hat{Q}_{\text{opt}}$ that satisfies

$$s_2^\infty \begin{pmatrix} R_{11} - \hat{Q}_{\text{opt}} \\ R_{21} \end{pmatrix} = 1.5897 < 1.6000$$

has the McMillan degree equal to 9, too. The first two singular value plots of

$$\begin{pmatrix} R_{11} - \hat{Q}_{\text{opt}} \\ R_{21} \end{pmatrix}$$

are also shown in Fig. 3.

The eigenvalues of G, K_{opt}, and $\hat{K}_{\text{opt}}$ are tabulated in Table 2. There are two unstable modes in K_{opt}, i.e., K_{opt} is unstable to stabilize the helicopter. The comparison of the second singular value plots of

$$\begin{pmatrix} R_{11} - Q_{\text{opt}} \\ R_{21} \end{pmatrix}$$

and

$$\begin{pmatrix} R_{11} - \hat{Q}_{\text{opt}} \\ R_{21} \end{pmatrix}$$

is shown in Fig. 3. It is explicit that the second singular value of the sublayers of

$$\begin{pmatrix} R_{11} - \hat{Q}_{\text{opt}} \\ R_{21} \end{pmatrix}$$

is always superior to that of

$$\begin{pmatrix} R_{11} - Q_{\text{opt}} \\ R_{21} \end{pmatrix}$$

in all frequencies without affecting the optimality of the major layer. The McMillan degree of the optimal controller $\hat{K}_{\text{opt}}$ whose state-space description is tabulated in Table 4 corre-

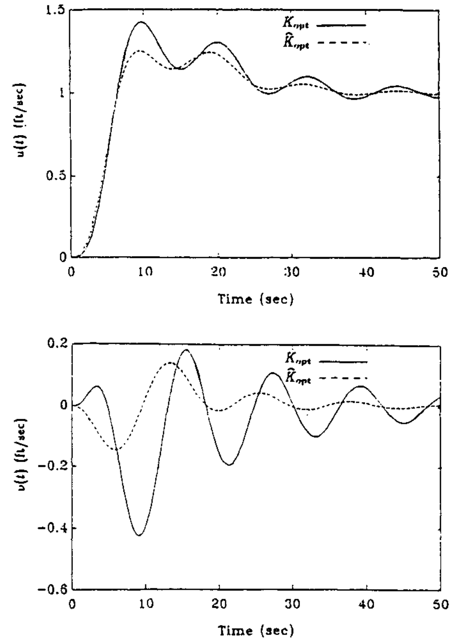

Fig. 5 Time responses of $u(t)$ and $v(t)$ with unit forward velocity command.

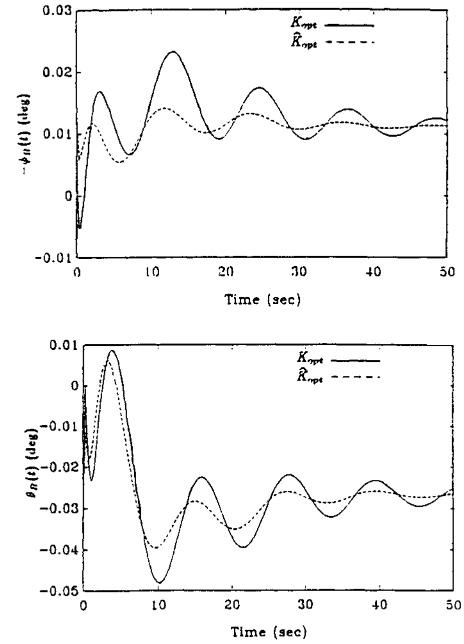

Fig. 7 Time responses of control signals with unit forward velocity command.

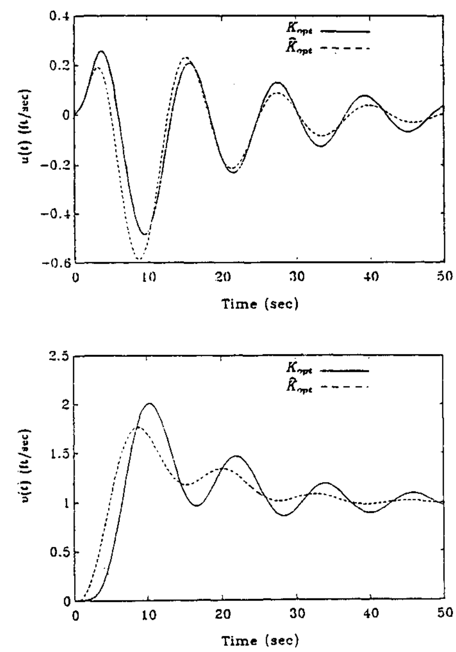

Fig. 6 Time responses of $u(t)$ and $v(t)$ with unit side velocity command.

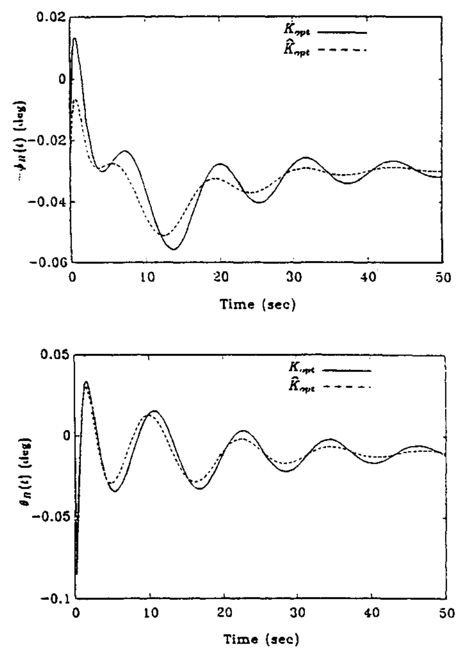

Fig. 8 Time responses of control signals with unit side velocity command.

sponding to $\hat{Q}_{opt}$ is also 5. The MIMO gain margin and phase margin[6, 23-25] can be expressed as follows:

$$\text{Gain margin} = 20 \mid \log(1 \pm MSV) \mid$$

$$\text{Phase margin} = \pm\cos^{-1}[1 - (MSV)^2/2]$$

The minimum singular values (MSV) of the return difference matrix $(I + GK)$, the gain margin, and phase margin are listed in Table 5. The truth, which is shown in Table 5, is that all performances of the closed-loop system with the optimal controller $\hat{K}_{opt}$ are better than that with K_{opt} in frequency domain and that these two controllers have the same McMillan degree is one of the most important results to apply the approach proposed in this paper.

In Fig. 4, the output complementary sensitivity function T is the transfer function from v to y_T, the output sensitivity function S is the transfer function for v to y_s, $r(t)$ is the command signal input, and $v(t)$ is the sensor noise inputs. Transient responses to the pilot command for the linearized model of this helicopter with optimal controller K_{opt} and $\hat{K}_{opt}$ are given in Figs. 5–8. Figures 5 and 7 illustrate a pure forward velocity maneuver resulting from step pilot longitudinal input of 1 ft/s, whereas Figs. 6 and 8 diagram a pure side velocity maneuver resulting from step pilot lateral input of 1 ft/s. In these figures, the solid lines are the responses by the controller K_{opt}, and the dash lines are those by the controller $\hat{K}_{opt}$. The overshoot of the responses by the controller $\hat{K}_{opt}$ are smaller than those by controller K_{opt}, especially the control signals θ_R and ϕ_R. See Figs. 7 and 8. These results show that $\hat{K}_{opt}$ reduces

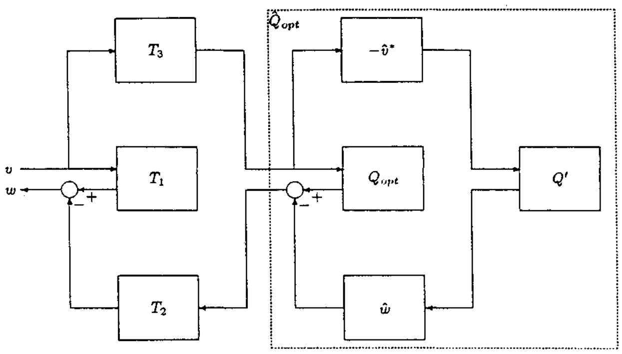

Fig. 9 Configuration of refined H^∞-optimal controller.

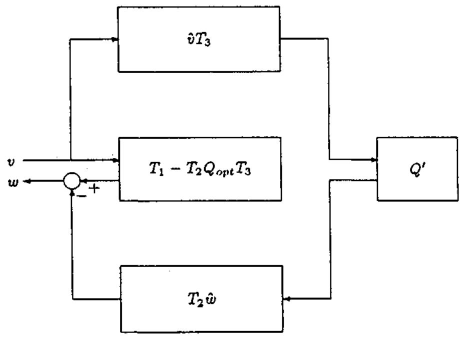

Fig. 10 Equivalent model matching problem of the sublayers.

the probability of the saturation effects in actuator comparing with K_{opt}. On the other hand, the settling times of all of the states of the plant $G(s)$ and the control signal u_c by $\hat{K}_{opt}$ are shorter than those by K_{opt}, i.e., the damping effect is more significant by $\hat{K}_{opt}$ than by K_{opt}. In other words, $\hat{K}_{opt}$ absorbs the energy excited from the plant as well as the exogenous input, including the sensor noise, much faster than K_{opt} does in all input-output channels. See Figs. 5-8.

In summary, the H^∞ norm of the sublayers can be improved by regulated $Q' \epsilon RH_+^\infty$. Figure 9 illustrates the framework of $\hat{Q}_{opt}$ with Q_{opt} and Q'. Figure 10, which is equivalent to Fig. 9, shows the standard general distance problem in H^∞ of the sublayers with any given $Q_{opt} \in \Xi$. It is explicit that the H^∞ norm of Eq. (18), which is the second singular value of Eq. (4), is under domination.

Conclusion

An approach considering the refined H^∞-optimal control has been developed in this paper. The diagonalizing matrix pair, the characterization of the sublayers, and the parameterization of all optimal solutions are fulfilled for the general distance problem in H^∞ entirely. From this proposed approach, the H^∞ norm of the sublayers can be both analyzed and synthesized by a parameter that is also a suboptimal solution of another four-block problem.

An engineering application is also demonstrated. The promising simulation results show that the refined H^∞-optimal control can improve the conventional one both in time domain and in frequency domain in the rotorcraft flight control without the inflation of the McMillan degree of the controller. It can be concluded that the performance in frequency domain is better by this proposed method. However, it cannot be asserted that the performance in time domain is better in all cases by this method since there is no direct testimony to prove these results. Nevertheless, it is sure in many cases due to our experience.

Appendix A

First, from Eqs. (5) and (7), it becomes

$$w_i^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v(s)$$

$$= w_i^T(s)\, w_i(-s)\, w_i^T(-s)\, w_c(-s)$$

or equivalently

$$w_i^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v_i(s)$$

$$= w_o(-s) w_c(-s) v_o^{-1}(s) v_c^{-1}(s)$$

$$= g(s)\, w_c(-s)\, v_c^{-1}(s) \tag{A1}$$

since $v^*v = w^*w$.[17] Both $v_c(s)$ and $w_c(-s)$ are all-pass scalar functions. Without loss of generality, $w_c(-s)v_c^{-1}(s)$ in Eq. (A1) can be absorbed by $g(s)$, or $g(s)\, w_c(-s)v_c^{-1}(s) \rightarrow g(s)$. Therefore,

$$w_i^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v_i(s) = g(s)$$

Second, calculate the part

$$w_\perp^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v_i(s)$$

It is easy to show that the part is equal to zero by Eq. (5). Also, the part

$$w_i^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v_\perp(s)$$

needs to be calculated. From Eq. (6), it is obvious that

$$w^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} v_\perp = v_c^* v_o^* v_i^* v_\perp$$
$$= 0$$

The part is equal to zero, and V and W are the diagonalizing matrix pair by the definition in Ref. 19. This completes the proof of proposition 3. □

Appendix B

For any two different optimal solutions Q_{opt} and $\hat{Q}_{opt}$, it is not difficult to show that

$$W^T(s)\begin{bmatrix} R_{11} - \hat{Q}_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} V(s)$$

$$= W_i^T(s)\begin{bmatrix} R_{11} - Q_{opt} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} V(s) - W^T(s)$$

$$\times \begin{bmatrix} \hat{Q}_{opt} - Q_{opt} & 0 \\ 0 & 0 \end{bmatrix} V(s) \tag{B1}$$

Equation (B1) yields

$$W^T(s)\begin{bmatrix} R_{11}-\hat{Q}_{\text{opt}} & R_{12} \\ R_{21} & R_{22}\end{bmatrix} V(s) = \begin{bmatrix} g(s) & 0 \\ 0 & \begin{bmatrix} \bar{R}'_{11} & \bar{R}'_{12} \\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix}\end{bmatrix} - \begin{bmatrix} w_{i,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{i,1} & \xi^* w_{i,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{\perp,1} \\ \xi^* w_{\perp,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{i,1} & \xi^* w_{\perp,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{\perp,1}\eta^*\end{bmatrix} \tag{B2}$$

Since

$$v_{\perp,1}^* v_{i,1} = -v_{\perp,2}^* v_{i,2} \tag{B3}$$

$$v_{\perp,1}^* v_{\perp,1} = -v_{\perp,2}^* v_{\perp,2} + I \tag{B4}$$

$$w_{i,1}^T(s) w_{\perp,1}(-s) = -w_{i,2}^T(s) w_{\perp,2}(-s) \tag{B5}$$

$$w_{\perp,1}^T(s) w_{\perp,1}(-s) = -w_{\perp,2}^T(s) w_{\perp,2}(-s) + I \tag{B6}$$

with Eq. (12), it yields

$$\begin{aligned} &w_{i,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{i,1} \\ &= w_{i,1}^T w_{\perp,1}(-s)\xi_i Q'\eta_i v_{\perp,1}^* v_{i,1} \\ &= w_{i,2}^T w_{\perp,2}(-s)\xi_i Q'\eta_i v_{\perp,2}^* v_{i,2} \text{ [from Eqs. (B3) and (B5)]} \\ &= 0 \text{ [from Eqs. (8) and (9)]} \end{aligned}$$

$$\begin{aligned} &w_{i,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{\perp,1}\eta^* \\ &= w_{i,1}^T w_{\perp,1}(-s)\xi_i Q'\eta_i v_{\perp,1}^* v_{i,1}\eta^* \\ &= -w_{i,2} w_{\perp,2}(-s)\xi_i Q'\eta_i v_{\perp,1}^* v_{\perp,1}\eta^* \text{ [from Eq. (B5)]} \\ &= 0 \text{ [from Eq. (9)]} \end{aligned}$$

$$\begin{aligned} &\xi^* w_{\perp,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{i,1} \\ &= \xi^* w_{\perp,1}^T w_{\perp,1}(-s)\xi_i Q'\eta_i v_{\perp,1}^* v_{i,1} \\ &= -\xi^* w_{\perp,1}^T w_{\perp,1}(-s)\xi_i Q'\eta_i v_{\perp,2}^* v_{i,2} \text{ [from Eq. (B3)]} \\ &= 0 \text{ [from Eq. (8)]} \end{aligned}$$

$$\begin{aligned} &\xi^* w_{\perp,1}^T(\hat{Q}_{\text{opt}}-Q_{\text{opt}})v_{\perp,1}\eta^* \\ &= \xi^* w_{\perp,1}^T w_{\perp,1}(-s)\xi_i Q'\eta_i v_{\perp,1}^* v_{\perp,1}\eta^* \\ &= \xi^*[I - w_{\perp,2}^T w_{\perp,2}(-s)]\,\xi_i Q'\eta_i\,[I - v_{\perp,2}^* v_{\perp,2}]\eta^* \\ &\quad \text{[from Eqs. (B4) and (B6)]} \\ &= \begin{bmatrix} Q' & 0 \\ 0 & 0\end{bmatrix} \text{ [from Eqs. (8) and (9)]} \end{aligned}$$

Equation (B2) yields

$$W^T(s)\begin{bmatrix} R_{11}-\hat{Q}_{\text{opt}} & R_{12} \\ R_{21} & R_{22}\end{bmatrix} V(s) = \begin{bmatrix} g(s) & 0 \\ 0 & \begin{bmatrix} \bar{R}'_{11} & \bar{R}'_{12} \\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix}\end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \begin{bmatrix} Q' & 0 \\ 0 & 0\end{bmatrix}\end{bmatrix} \tag{B7}$$

From Eq. (B7), it can yield

$$W^T(s)\begin{bmatrix} R_{11}-\hat{Q}_{\text{opt}} & R_{12} \\ R_{21} & R_{22}\end{bmatrix} V(s) = \begin{bmatrix} g(s) & 0 & 0 \\ 0 & \bar{R}'_{11}-Q' & \bar{R}'_{12} \\ 0 & \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix}$$

$$s_1^\infty\begin{bmatrix} \bar{R}'_{11}-Q' & \bar{R}'_{12} \\ \bar{R}'_{21} & \bar{R}'_{22}\end{bmatrix} \le \gamma' \le 1$$

if and only if

$$s_2^\infty\begin{bmatrix} R_{11}-\hat{Q}_{\text{opt}} & R_{12} \\ R_{21} & R_{22}\end{bmatrix} \le \gamma'$$

From Eq. (12), $w_{\perp,1}(-s)\xi_i Q'\eta_i v_{\perp,1}^* \in RH_+^\infty$ because $\hat{Q}_{\text{opt}} \in RH_+^\infty$. This completes the proof of Theorem 4. □

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